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SCIENTIFIC COMMENT

Constraints on the degree of dynamic partial melting and source composition using concentration ratios in magmas

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Abstract—We present a dynamic melting inversion (DMI) method that permits calculation of the degree of partial melting for cogenetic primary magmas in the context of the dynamic melting model of Langmuir et al. (1977). The method uses variations of between-magma concentration ratios for two incompatible trace elements that have different bulk distribution coefficients, and does not require assumptions regarding source incompatible element concentrations or ratios. Source concentrations can be calculated after obtaining the partial melting degree. The method is applied to the nephelinitic Koloa suite of Kauai, the Honolulu volcanics of Oahu, and alkalic basalts from southeastern Australia. Results, which display remarkable intersuite and interelement consistency, suggest that melilitites and alkali olivine basalts are formed by 2–4% and 8–11% partial melting, respectively, of a LREE-enriched mantle source. This result for basalts that are characterized by positive ϵ_{Nd} argues persuasively that the often-invoked "recent source enrichment event" is actually an integral part of the total melting process.

1. INTRODUCTION

Through the study of mantle-derived basaltic magma, geochemists and petrologists attempt to place constraints on the nature of the melting process, the degree of partial melting, the magma extraction and aggregation process, values for partition coefficients between mantle minerals and basaltic magma, and on the chemical, isotopic, and mineralogical character of the mantle source. Unfortunately, due to the interdependence of these parameters and processes, if we want to learn about one of them, we must often make poorly founded assumptions about the others. Therefore, we are continually looking for ways to reduce the extent to which the problem is under-determined so as to increase the yield of model-independent conclusions that derive from individual investigations. The nature of this problem is nicely illustrated by considering the degree of partial melting that leads to the production of a particular basalt. In the context of a simple batch melting model, the degree of partial melting can only be estimated from trace element concentrations in the magma by assuming concentration levels in the source. This exercise is often problematic and can lead to significant error in the estimated degree of partial melting. For example, although heavy rare earth elements (HREEs) typically vary little between basalts, reflecting a relative constancy among mantle sources, literature estimates for basalt-source HREE abundances vary from about 1 to more than 7 times chondritic values (see, e.g., Clague and Frey, 1982; Loubet et al., 1975; Frey, 1969; Kay and Gast, 1973; throughout this paper, chondrite-normalized REE concentrations are reported relative to the average C1 values of Anders and Ebihara, 1982). Although interregional variations in mantle HREE levels un-

doubtedly exist, literature estimates are not uniquely defined and tend to be strongly model-dependent. It would be extremely useful to be able to unequivocally estimate the degree of partial melting for primary magmas without having to make assumptions about source concentrations.

Two methods for calculating the approximate degree of partial melting without making assumptions about source concentrations have been reported. One method uses the concentration ratios of two incompatible trace elements in at least two magmas presumed to derive from the same source; it is called the concentration ratio (CR) method (Maaløe, 1994). The second method assumes concentration ratios in the source (Treuil and Joron, 1975; Minster and Allègre, 1978; Hofmann and Feigenson, 1983; Cebriá and López-Ruiz, 1995) and is called the source ratio (SR) method. These two methods are summarized and briefly reviewed in the Appendix. The CR method may be used for a range of melting models, whereas the SR method can only be applied for batch melting. Maaløe (1994) has shown that the CR method for (perfect) fractional melting does not yield meaningful partial melting degrees; for example, the calculated partial melting degree for alkali olivine basalt at Kauai is 26.6%, which is clearly too high regardless of the model context. The CR method for batch melting, and the SR method, are based on a batch melting model deduced from the classic equations of Shaw (1970). However, as our understanding of the physics of the melting process has evolved, workers in the field have come to generally agree that true batch melting does not, in general, occur during basalt production, and that dynamic melting, as proposed by Langmuir et al. (1977) and formulated by McKenzie (1985a) and Maaløe and Johnston (1986), is a more realistic scenario (e.g., Ribe, 1988; Hèmond et al., 1994). In the context of

dynamic melting, the batch melting approximation worsens as the degree of melting goes beyond the porosity limit of the system and melt begins to separate from the solid ($\sim 1\%$; McKenzie, 1985b; Ribe, 1985; Qin, 1993). The principle aim of this paper is to derive a new set of equations to more reliably determine the degree of partial melting and the source composition by using concentration ratios of two incompatible trace elements in two magmas of common parentage in the context of dynamic melting. We call this method the DMI, or dynamic melting inversion, method.

2. CALCULATION METHOD

We use the dynamic melting formulation developed by McKenzie (1985a) for the situation where the rate of melting and porosity are constant and finite while the system of matrix and interstitial fluid is moving. This requires that the melt in excess of the porosity be extracted from the matrix at the same rate at which it is formed (the details of the model are shown in Fig. 3 of McKenzie, 1985a). We let $\beta = \nu_e/\nu_m$ represent the ratio of melt extraction rate (ν_e) over melting rate (ν_m). For dynamic melting, β can be expressed in a step-function form (Qin, 1992) as follows:

$$\beta = 0, \quad \text{when} \quad f \le \phi \tag{1}$$

and

$$\beta = \frac{1}{1-\phi}, \quad \text{when} \quad f > \phi; \tag{2}$$

for batch melting,

$$\beta = 0, \quad \text{for all values of } f \text{ and } \phi,$$

and, for perfect fractional melting,

$$\beta = 1 \quad \text{and} \quad \phi = 0, \quad \text{for all values of } f.$$

ϕ is the threshold value for melt separation, or the porosity of the residual solid. During dynamic melting, when the melt fraction in the solid matrix is less than ϕ, there is no melt extraction (as in batch melting); when the melt fraction in the solid is greater than ϕ, melt will be extracted at about the rate at which it forms. ϕ is estimated to be $\sim 1\%$ for basaltic melt (McKenzie, 1985b; Qin, 1993). Note that for dynamic melting, the extraction rate is actually higher than the melting rate after the threshold melt fraction is reached. This is because the volume of solid matrix is decreasing during melting; thus, to keep the melt fraction in the matrix constant, a small amount of extra melt has to be extracted (Qin, 1992). Since f represents a mass fraction while ϕ is a volume fraction, we can replace ϕ in Eqns. 1 and 2 by f_0, the mass fraction of liquid present just as extraction begins, by using the relationship between porosity and mass fraction given by

$$f_0 = \frac{\rho_f \phi}{\rho_f \phi + \rho_s (1 - \phi)}. \tag{3}$$

Then, for dynamic melting, β can be expressed as follows:

$$\beta = 0, \quad \text{when} \quad f \le f_0, \tag{4}$$

$$\beta = \frac{1}{1-\phi}, \quad \text{when} \quad f > f_0. \tag{5}$$

For batch melting,

$$\beta = 0, \quad \text{for all values of } f \text{ and } f_0,$$

and for fractional melting,

$$\beta = 1 \quad \text{and} \quad f_0 = \phi = 0, \quad \text{for all values of } f.$$

The concentrations in the residual melts (C_r) of the dynamic melting model are given by McKenzie (1985a) as

$$C_r = GC_0(1 - X)^{G(1-D)}, \tag{6}$$

where

$$G = \frac{\rho_f \phi + \rho_s (1 - \phi)}{\rho_f \phi + \rho_s (1 - \phi)D}; \tag{7}$$

C_0 is the initial concentration of the element in the source, D is the bulk distribution coefficient, ρ_f is the density of melt (~ 2.8 g/cm³ for basaltic melt), and ρ_s is the density of solid matrix (~ 3.3 g/cm³ for peridotite).

The concentration of a trace element in the extracted dynamic melt (C_L) can be obtained by integrating Eqn. 6 from 0 to X, where X is the mass fraction of liquid extracted, and then dividing by X:

$$C_L = \frac{1}{X} C_0 G \frac{1 - (1 - X)^{G(1-D)+1}}{G(1-D) + 1}. \tag{8}$$

Let the degree of partial melting (f) increase from stage 1 (f_1) to stage 2 (f_2), while the mass fraction of liquid extracted increases from X_1 to X_2. The enrichment ratio Q for the highly incompatible element (e.g., Th, Rb, P, or Ba) is

$$Q_a = \frac{C_a^1}{C_a^2} = \frac{X_2[1 - (1 - X_1)^{G_a(1-D_a)+1}]}{X_1[1 - (1 - X_2)^{G_a(1-D_a)+1}]}. \tag{9}$$

Similarly, for the less incompatible element (e.g., an REE),

$$Q_b = \frac{C_b^1}{C_b^2} = \frac{X_2[1 - (1 - X_1)^{G_b(1-D_b)+1}]}{X_1[1 - (1 - X_2)^{G_b(1-D_b)+1}]}. \tag{10}$$

We select both highly incompatible and not-so-highly incompatible elements because they have large but different enrichment ratios (Q) in magmas formed at different degrees of partial melting. The important feature of Q_a and Q_b is that both of them are independent of the source concentration (C_0). Equations 9 and 10 constitute a set of nonlinear equations. They have only two unknowns, X_1 and X_2, and can be solved by Newton's method for a system of nonlinear equations. A detailed FORTRAN program written to solve these equations may be obtained from the authors. After obtaining X_1 and X_2, the degree of partial melting can be calculated as follows:

$$f = X \frac{\rho_s(1 - \phi)}{\rho_f \phi + \rho_s(1 - \phi)} + \frac{\rho_f \phi}{\rho_f \phi + \rho_s(1 - \phi)}, \tag{11}$$

where the first and second terms in Eqn. 11 represent the mass fraction of extracted liquid and residual liquid, respectively. The source concentrations can then be calculated from the relationship

$$C_0 = \frac{C_L X[G(1 - D) + 1]}{G[1 - (1 - X)^{G(1-D)+1}]}. \tag{12}$$

2.1. Sensitivity of the DMI Method to Variations in D and ϕ

The sensitivity of the extent of partial melting and source composition, as predicted by the DMI method, to variations in D and ϕ is assessed in Fig. 1. The calculations are made for the Koala melilitite and alkali basalt, using Nd for Q_b. Reported D values for the REEs in olivine, diopside, enstatite, and pyrope typically vary with a factor from 2 to 4 for each of the elements (see, e.g., Frey et al., 1978; Irving, 1978; Minster and Allègre, 1978; Leeman et al., 1980; Chen et al., 1990). As seen in Fig. 1a,b, which show a range in D_{Nd} of a factor of 3, the effect of variation in D_{Nd} on both f_1 and $(C_0)_{Nd}$ is somewhat demagnified by the calculation; that is, a 40% change in D_{Nd} leads to about a 30% change in either f_1 or $(C_0)_{Nd}$.

The effects of varying ϕ on f_1 and $(C_0)_{Nd}$ are shown in Fig. 1c,d. Here, we see that, in fact, both f_1 and $(C_0)_{Nd}$ are rather
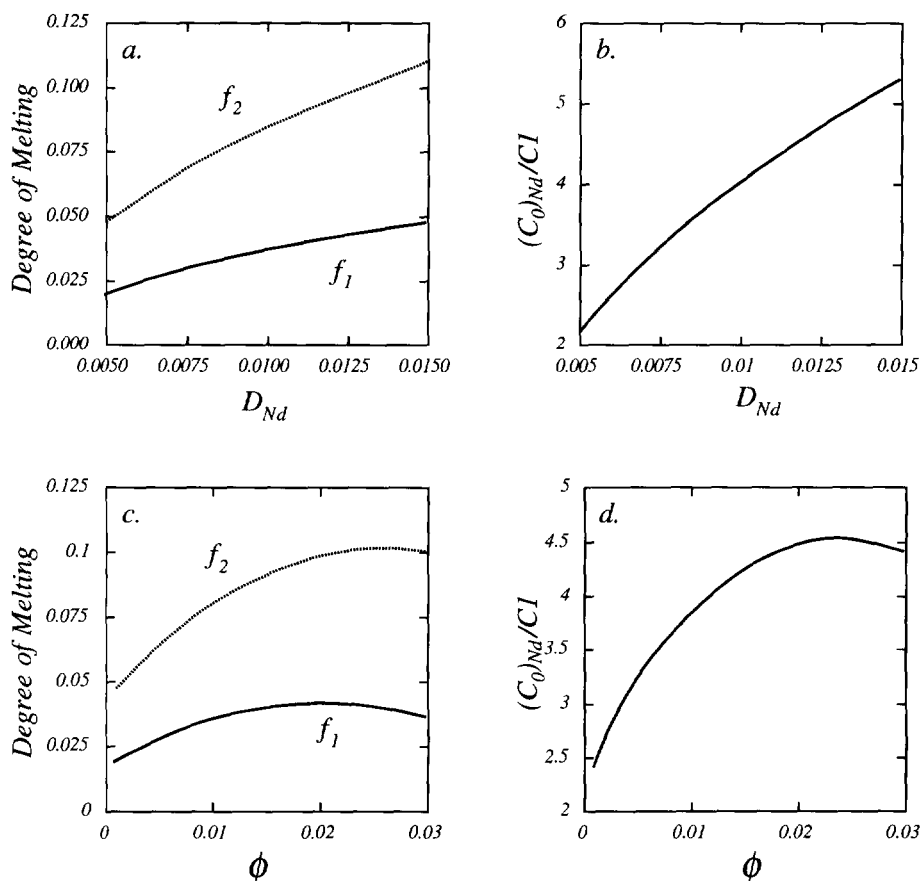
Fig. 1. Variation diagrams documenting the sensitivity of $(C_0)_{Nd}$ and f to uncertainties in D and ϕ. The curves in (a) and (b) were obtained by setting $\phi = 0.01$. The curves in (c) and (d) were obtained by setting $D_{Nd} = 0.0095$. See text for explanation. f_1 and f_2 are the degrees of partial melting for the melilitite and alkali olivine basalt from Koloa. $(C_0)_{Nd}/$ Cl is the Cl chondrite-normalized source Nd concentration.

insensitive to variations in ϕ. A change in ϕ from 0.01 to 0.02, results in only about a 20% change in f_1 and $(C_0)_{Nd}$.

3. APPLICATIONS AND DISCUSSION

3.1. Application to Hawaiian and Southeastern Australian Basalts

The application of the DMI method can be demonstrated using trace element data for suites of alkalic volcanics from Hawaii and southeastern Australia. Data and results for the nephelinitic Koloa suite of Kauai, Hawaii, USA, are summarized in Table 1 (all relevant parameters for this calculation are given in Table 1; data are from Maaløe et al., 1992; distribution coefficients are from Maaløe, 1994). The composition of the primary basalts ranges from melilitite to alkali olivine basalt, and the variation in P_2O_5 content is from 0.77 to 0.29%. Q_a is taken as the P_2O_5 concentration ratio (melilitite/alkali olivine basalt), and the Nd concentration ratio is used to derive Q_b for this suite:

$$Q_a = Q_{P_2O_5} = 0.77/0.29 = 2.66 \qquad (13)$$

and

$$Q_b = Q_{Nd} = 52.1/23.5 = 2.21. \qquad (14)$$

Using these enrichment ratios and solving Eqns. 9 and 10, we obtain $X_1 = 2.6\%$ and $X_2 = 7.3\%$. Substituting these values into Eqn. 11, we obtain $f_1 = 3.5\%$ for the melilitite, and $f_2 = 8.2\%$ for the alkali olivine basalt. Similarly, using the P_2O_5 concentration ratio for Q_a, but La or Ce (instead of Nd) to obtain Q_b, we can calculate two additional sets of f_1 and f_2 values; the resulting variations in f_1 and f_2 range from 3.4 to 4.0%, and from 8.0 to 9.5%, respectively (Table 1). Although a lack of strict adherence to the dynamic melting model can certainly be expected to result in some variation in these f values, the real uncertainty in the bulk distribution coefficients is more than sufficient to explain the observed ranges in f_1 and f_2. Averaging the values obtained for the three light rare earth elements (LREEs) yields $f_1 = 3.6\%$, $X_1 = 2.7\%$, $f_2 = 8.6\%$, and $X_2 = 7.7\%$. Using these average values for X_1 or X_2, and the trace element concentrations in the melilitite or alkali olivine basalt, Eqn. 12 allows us to calculate source concentrations of 1.40, 2.86, and 1.82 ppm for La, Ce, and Nd, respectively.

By comparison, using CR method for batch melting, Maaløe (1994) estimated f_1 to be 1.1, 2.1, or 6.3%, using enrichment ratios for La, Ce, and Nd, respectively. The average f_1 calculated by this method is 3.2%, similar to the result

Table 1. Estimation of partial melting degrees and mantle source composition for melilitite and alkali olivine basalt of the nephelinitic Koloa volcanics

Element	D	Melilitite (n=5)	Alkali (n=3)	Q	f_1 (%)	f_2 (%)	C_0	$(C_0)_N$
P₂O₅	0.001	0.77%	0.29%	2.66				
La	0.0021	47.1	18.1	2.60	3.4	8.0	1.40	5.9
Ce	0.0041	92.4	36.4	2.54	4.0	9.5	2.86	4.6
Nd	0.0095	52.1	23.5	2.21	3.5	8.2	1.82	4.0
Ave_DMI							3.6	8.6
Ave_BM							3.2	7.9
Ave_DMI-Ave_BM							0.4	0.7

La, Ce, Nd (ppm) and P₂O₅ (%) concentrations are from Maaløe et al. (1992).
Ave_DMI = the average of partial melting degrees from CR method for dynamic melting;
Ave_BM = the average of partial melting degrees from CR method for batch melting; Ave_BM is given here for comparison.
D = bulk distribution coefficient; Q = enrichment concentration ratio = concentration in melilitite/concentration in alkali basalt; f_1 = partial melting degree for melilitite; f_2 = partial melting degree for alkali olivine basalt; C_0 = source concentration; $(C_0)_N$ = C1 chondrite-normalized source concentration; n = number of samples.
P₂O₅ concentration ratio is used as Q_a (the enrichment ratio for the highly incompatible element) and REE ratio as Q_b (the enrichment ratio for the less incompatible element). The REE distribution coefficients of set 1 by Frey et al. (1978) are used in the calculations. The mineralogical proportions can be considered similar to that of average lherzolite: olivine: 66%, enstatite: 20%, diopside: 8%, and pyrope: 6% (Maaløe and Aoki, 1977; Maaløe, 1994). REE abundances for C1 chondrites are from Anders and Ebihara (1982). Other parameters chosen: $D_{P2O5} = D_{Th} = 0.001$ (Maaløe, 1994), $\phi = 1\%$, $\rho_f = 2.8$ g/cm³, and $\rho_s = 3.3$ g/cm³.

obtained here (Table 1), but with a much larger associated uncertainty (total range of 5.2% compared to 0.6% for our method).

Using Eqns. 9, 10, and 11, and the Th and REE data of Clague and Frey (1982) for the Honolulu volcanics, Oahu, Hawaii, USA, we obtain partial melting degrees of 2.1% for melilitite and 8.5% for alkali olivine basalt (these melting degrees represent averages for calculations made with Q_bs for eight different REEs, ranging from La to Lu; Table 2). Clague and Frey (1982) suggested that the alkali olivine basalt was formed by a relatively large degree of melting (>5.5%), and our results for the DMI method support that view. Our calculation also supports the contention of Maaløe et al. (1992) that melilitites of the Honolulu volcanics formed by smaller degrees of partial melting than the Koloa melilitites, and that the difference in their respective source compositions is small.

Trace element data from Frey et al. (1978) for basalts from southeast Australia yield average partial melting degrees of 3.2% for olivine melilitite and 10.7% for alkali olivine basalt (these melting degrees also represent averages for calculations made with Q_bs for eight different REEs, ranging from La to Lu; Table 3). Our model source REE pattern for southeastern Australia (Table 3, Fig. 2) is concave upward, and has REE concentrations that range from 3 to 10 times C1 chondrites. In the context of a source with chondritic-relative REE abundances at 2.5 to 6.3 times C1 levels, Frey et al. (1978) showed that an extremely small degree of melting is required to produce the olivine melilitite (0.4%). Values of f

Table 2. Estimation of partial melting degrees and mantle source composition for melilitite and alkali olivine basalt from the Honolulu volcanics

Element	D	Melilitite (n=7)	Alkali (n=2)	Q	f_1 (%)	f_2 (%)	C_0	$(C_0)_N$
Th	0.001	9.6	2.1	4.57				
La	0.0021	90.1	21	4.29	1.9	7.7	1.59	6.7
Ce	0.0041	160	44	3.64	1.2	6.0	3.11	5.0
Nd	0.0095	66.7	22	3.03	1.8	7.7	1.63	3.6
Sm	0.0180	15.2	5.91	2.57	2.3	9.2	0.50	3.4
Eu	0.0228	4.49	1.98	2.27	2.2	8.8	0.17	3.0
Tb	0.0330	1.64	0.81	2.02	2.5	9.7	0.08	2.3
Yb	0.2644	1.8	1.6	1.13	2.8	11.0	0.50	3.1
Lu	0.4460	0.26	0.25	1.04	2.0	8.2	0.12	4.8
Ave_DMI							2.1	8.5
Ave_BM							1.1	4.9
Ave_DMI-Ave_BM							1.0	3.6

Trace element concentrations (ppm) are from Clague and Frey (1982).
D, Q, f_1, f_2, C_0, $(C_0)_N$, n, Ave_DMI, Ave_BM and other parameters chosen are the same as in Table 1. Th concentration ratio is used as Q_a and REE ratio as Q_b.

Table 3. Estimation of partial melting degrees and mantle source composition for melilitite and alkali olivine basalt from southeastern Australia

Element	D	Melilitite (n=2)	Alkali (n=1)	Q	f_1 (%)	f_2 (%)	C_0	$(C_0)_N$
Th	0.001	11.5	2.9	3.97				
La	0.0029	89	23	3.87	3.3	11.1	2.28	9.7
Ce	0.0041	170	49	3.47	2.3	8.0	4.60	7.5
Nd	0.0095	70.5	23	3.07	2.9	9.9	2.21	4.8
Sm	0.0180	15.2	5.53	2.75	3.6	12.1	0.59	4.0
Eu	0.0228	4.8	2.02	2.38	3.2	10.8	0.21	3.8
Tb	0.0330	2.0	0.87	2.30	4.0	13.6	0.11	3.1
Yb	0.2644	1.9	1.7	1.12	3.4	11.6	0.53	3.3
Lu	0.4460	0.24	0.23	1.04	2.5	8.7	0.11	4.5
Ave_DMI							3.2	10.7
Ave_BM							2.1	7.5
Ave_DMI-Ave_BM							1.1	3.2

Trace element concentrations (ppm) are from Frey et al.(1978).
D, Q, f_1, f_2, C_0, $(C_0)_N$, n, Ave_DMI, Ave_BM and other parameters chosen are the same as in Table 1. Th concentration ratio is used as Q_a and REE ratio as Q_b.

significantly less than 1% are considered unlikely because of the difficulty associated with the extraction of very small amounts of melt from the source (Richter and McKenzie, 1984; Ribe, 1988) due to surface forces that inhibit magma segregation until appreciable extents of melting (≥1%) have been achieved (Jurewicz and Watson, 1985). In their preferred model, Frey et al. (1978) estimated a partial melting degree of 4–6% for olivine melilitite and 11–15% for alkali olivine basalt, in the context of a LREE-enriched source, ranging from 7.9 to 11.8 times C1 for the LREEs, to 3.1 to 3.8 times C1 for the HREEs. These results are in reasonably good agreement with those of our DMI method.

Average results for f_1 and f_2 from the CR method for batch melting are also shown in Tables 1–3. The CR method for batch melting tends to produce results that are more consistent with those of the DMI method at lower degrees of melting, in accord with the assessment of Maaløe (1994).

3.2. Mantle Source Characteristics and Implications for the "Total" Melting Process

Calculated REE concentrations for the mantle sources of the Koloa volcanics, Honolulu volcanics, and the southeastern
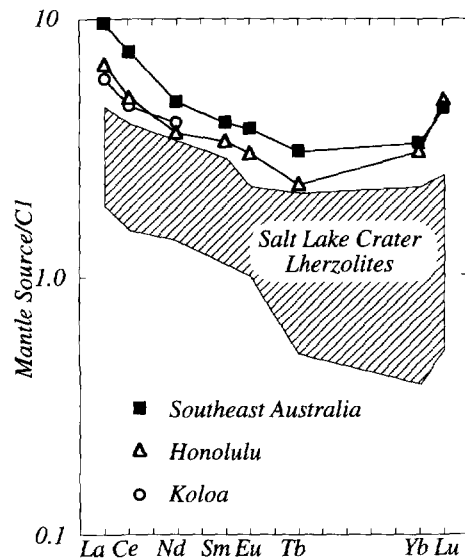

FIG. 2. C1 chondrite-normalized source REE compositions for melilitites and alkali olivine basalts from Koloa, Honolulu, USA, and southeastern Australia. The hatched region is the range for Salt Lake Crater lherzolites from Frey and Roden (1987). C1 normalizing values are from Anders and Ebihara (1982).

Australian basalts are given in Tables 1-3, respectively. The model mantle source of the southeastern Australian basalts has higher LREE abundances than the sources of the Koloa and Honolulu volcanics (Fig. 2), although the three sources all display similar LREE enrichments. Neodymium isotope ratios, however, suggest a LREE-depleted source for all three of these suites ($\epsilon_{Nd} > 0$; O'Nions et al., 1977; McDonough et al., 1985; Maaløe et al., 1992). This apparently incongruous view of the mantle source for alkalic melts is more the rule than the exception, as many oceanic and continental alkalic suites have positive ϵ_{Nd} in association with strong LREE enrichments. As has often been pointed out, these observations can be reconciled by proposing the long-term existence of a LREE-depleted source that only recently becomes enriched (as suggested by the convex-upward shapes of the model source REE patterns shown in Fig. 2), and subsequently melts to produce the LREE-enriched lavas. Although this sort of interpretation has essentially attained the status of "geochemical conventional wisdom," we consider it rather improbable that source enrichment, which is *unrelated* to the later melting event that produces the lavas, should so ubiquitously affect alkali-basalt source regions. Thus, it seems likely that the enrichment is actually an integral part of the melting event. In this context, one may consider that the enrichment documents an "autometasomatic event" (in the sense of Roden et al., 1984) as CO_2-rich fluids are released in the subsolidus peridotite shortly before the melting, in the conventional sense, takes place.

We envision an integrated melting process, then, where the first stage affects a large volume of source material (O'Hara, 1985; McKenzie, 1985b) and produces, at or just prior to the "true" onset of melting, a low-viscosity, LREE-enriched CO_2-rich magma or silica-rich fluid. This CO_2-rich fluid or melt, produced in the peripheral regions of a melting anomaly, will separate from its source and invade the overlying or adjacent mantle to produce a "second-stage" source which is ultimately affected by greater extents of melting (see, e.g., Graham et al., 1988; Zindler and Jagoutz, 1988). The ϕ value for the first-stage process should be less than 0.01 (which is the ϕ value for the second stage that is well described by the dynamic melting model) because of the very low viscosity of the CO_2-rich fluid or melt, compared with basaltic magma (McKenzie, 1985b; Minarik and Watson, 1995). Thus, the first stage of the integrated melting process, as envisioned here, is characterized by a smaller ϕ value, smaller degree of "melting," but larger melting volume, compared to the second stage, which corresponds to the "conventional" melting event.

REE characteristics of lherzolite xenoliths from Salt Lake Crater (Frey and Roden, 1987), one of the eruption sites of the Honolulu volcanics, are compared to those of our model Hawaiian sources in Fig. 2. While our estimated source compositions do not strictly fall within the range measured for the lherzolites, the shapes of the overall patterns are similar and could be reconciled with one another if relatively minor amounts of olivine fractionation have contributed to an overall enrichment of trace element abundances in the erupted magmas. Despite their enrichment in LREEs, the Salt Lake Crater lherzolites have MORB-like $^{143}Nd/^{144}Nd$ ratios (ϵ_{Nd} from about +8 to +9; Salters and Zindler, 1995). Thus, these

potential source rocks for the Honolulu volcanics (we note that Salters and Zindler, 1995, have shown that hafnium isotope systematics are *not* consistent with this genetic relationship) present a similar dilemma as do the magmas themselves, in that the LREE enrichment cannot be a long-term feature. We might envision, then, that lherzolites such as these represent a LREE-enriched, stage-two source that has already been modified by stage one of the melting process. LREE-enriched and -depleted examples of similar lherzolitic compositions (pre- and post-stage one) often occur in xenolith suites (see, e.g., Zindler and Jagoutz, 1988).

Although typical melting models, and, indeed, the dynamic melting model on which our DMI method is based, apply only to the second or final stage of the melting event, we believe that important future discoveries may hinge on eventual development of quantitative models for the complete melting process.

4. CONCLUSIONS

The dynamic melting inversion, or DMI, method presented here builds on the approach of Maaløe (1994), and enables calculation of the degree of partial melting and source composition for cogenetic primary mantle-derived magmas. The major difference between our method and those of Maaløe (1994) or Treuil and Joron (1975) is that ours is based on the dynamic melting model, instead of batch or fractional melting, and, therefore, works within what many would consider to be a more realistic melting context. Consequently, the results obtained are expected to more accurately reflect the melting process.

Application of the DMI method to several suites of alkalic volcanic rocks, the Koloa volcanics, the Honolulu volcanics, and basalts from southeastern Australia, suggests that melilitites are formed by 2-4% partial melting, and alkali olivine basalts represent 8-11% partial melts, of a LREE-enriched mantle source. The viability of the method is supported by the observed intersuite consistency and the high level of agreement obtained by using different element pairs for the same basalts. Our results, in accord with numerous previous studies, quantitatively document the derivation of basalts with positive ϵ_{Nd} from LREE-enriched sources. The near-ubiquity of this result for alkalic suites argues that the often-proposed "recent source enrichment" occurs as an integral, initial phase of the total melting process. Although our method does not address this initial melting phase, we believe that future progress in quantifying the production and sources of mantle-derived basalts will depend on efforts that do so.

The DMI method works very well for cogenetic pairs of primary alkalic basalts. Further work may extend application of the method to different kinds of basalts produced in different tectonic settings, given that observed or inferred primary magma compositions exhibit sufficient variation in trace element concentrations.

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APPENDIX

Concentration Ratio (CR) Method for Batch Melting

In the context of batch melting, the variation in the concentration of a trace element, c, in a melt is given by

$$c = \frac{c_0}{D + f(1 - P)}, \tag{A1}$$

where c_0 is the concentration in the source, D the distribution coefficient, f the degree of partial melting, and P is a constant whose value is determined by the proportions in which the minerals contribute to the melt (Shaw, 1970). For elements a (highly incompatible) and b (not-so-highly incompatible), their concentrations in the melt, c_a and c_b, respectively, are given by

$$c_a = \frac{c_a^0}{D_a + f(1 - P_a)} \tag{A2}$$

and

$$c_b = \frac{c_b^0}{D_b + f(1 - P_b)}. \tag{A3}$$

We let the degree of partial melting increase from stage 1 to stage 2. Using Eqn. A2 for two different degrees of partial melting, f_1 and f_2,

the enrichment ratio, Q_a, of the highly incompatible element a is given by

$$Q_a = \frac{c_a^1}{c_a^2} = \frac{D_a + f_2(1 - P_a)}{D_a + f_1(1 - P_a)}.$$ (A4)

Similarly, using Eqn. A3, the enrichment ratio, Q_b, for the less incompatible element is defined by

$$Q_b = \frac{c_b^1}{c_b^2} = \frac{D_b + f_2(1 - P_b)}{D_b + f_1(1 - P_b)}.$$ (A5)

Equations A2 to A5, above, are from Maaløe (1994), who showed that f_1 and f_2 can be obtained from Eqns. A4 and A5 graphically by using f vs. Q_b diagrams (see, p. 2520, Maaløe, 1994). These diagrams are rather cumbersome, however, as each applies for only a single value of Q_a, and explicit solutions can be obtained as follows:

$$f_1 = \frac{D_a(1 - P_b)(1 - Q_a) + D_b(1 - P_a)(Q_b - 1)}{(Q_a - Q_b)(1 - P_a)(1 - P_b)},$$ (A6)

$$f_2 = \frac{Q_b(D_b + f_1(1 - P_b)) - D_b}{1 - P_b}.$$ (A7)

The values for f_1 and f_2 given by these expressions may yield fairly reliable results for primary magmas formed at low degrees of partial melting. In comparison, the system of Eqns. 9 and 10 for dynamic melting do not yield explicit solutions and must be solved numerically. The calculated values for f_1 and f_2 from Eqns. A6 and A7 can be used to obtain good initial estimates for the numerical solution.

Source Ratio (SR) Method

Treuil and Joron (1975) and Minster and Allègre (1978) derived the following linear equation for batch melting:

$$\frac{c_a}{c_b} = \left[D_b - \frac{(1 - P_b)D_a}{1 - P_a} \right] \frac{c_a}{c_b^0} + \frac{(1 - P_b)c_a^0}{(1 - P_a)c_b^0}.$$ (A8)

Using c_a/c_b vs. c_a diagrams, the values of the slope (S) and the intercept (I) can be obtained from Eqn. A8, where

$$S = \left[D_b - \frac{(1 - P_b)D_a}{1 - P_a} \right] \frac{1}{c_b^0}$$ (A9)

and

$$I = \frac{(1 - P_b)c_a^0}{(1 - P_a)c_b^0}.$$ (A10)

At $c_a = 0$, if P_a and P_b are small or of similar value, then

$$I = \frac{c_a^0}{c_b^0}.$$ (A11)

Thus, the approximate source concentration ratio is obtained from the intercept, I, and this can then be used to calculate f. From Eqns. A2 and A3,

$$\frac{c_a^0}{c_b^0} = I = \frac{c_a[D_a + f(1 - P_a)]}{c_b[D_b + f(1 - P_b)]}.$$ (A12)

Defining

$$R = \frac{c_a}{c_b},$$ (A13)

we obtain

$$f = \frac{RD_a - ID_b}{I(1 - P_b) - R(1 - P_a)}.$$ (A14)

Equations A8 to A14 describe the SR method of Treuil and Joron (1975) and Minster and Allègre (1978). The approach may be af-

fected by the approximation at Eqn. A11 and the uncertainty of the intercept and the slope. This uncertainty results from the fact that c_a/c_b vs. c_a diagrams often define poor linear relationships (Clague and Frey, 1982; Gianetti and Ellam, 1994), in part because the trace element concentrations deviate from the predictions of the batch melting model.

For a highly incompatible element, a, assuming $D_a = P_a = 0$, from Eqns. A9 and A10, we have

$$S = \frac{D_b}{c_b^0},$$ (A15)

$$I = \frac{(1 - P_b)c_a^0}{c_b^0},$$ (A16)

and Eqn. A8 becomes

$$\frac{c_a}{c_b} = c_a S + I,$$ (A17)

where S and I are the slope and intercept of the straight line described by Eqn. A17. By rearranging Eqn. A16, we obtain

$$\frac{c_b^0}{c_a^0} = \frac{1 - P_b}{I},$$ (A18)

and by rearranging Eqns. A15 and A16, we obtain

$$\frac{D_b}{c_a^0} = \frac{S(1 - P_b)}{I}.$$ (A19)

Equations A15 and A19 are from Hofmann and Feigenson (1983). This system of equations does not yield absolute source concentrations and degrees of partial melting, but can be used to obtain source concentrations and distribution coefficients *relative* to the source concentration of element a.

Recently, Cebriá and López-Ruiz (1995) proposed a method to calculate mantle source concentrations, bulk distribution coefficients (D) and the constant P. They combined Eqns. A3, A9, and A10 to calculate c_a^0, D_b, and P_b for different elements after estimating f, c_a^0, D_a, and P_a. The advantage of this method is that it embodies fewer assumptions about the mineralogical proportions of the source mantle and is, therefore, independent of the choice of D values for minerals. However, the resulting values of c_a^0, D_b, and P_b depend on the accuracy of f and c_a^0, which were estimated by setting $D_a = P_a = 0$, and assuming source concentrations for elements that show relatively little variation in the mantle (e.g., Yb and Lu). They established the following system of equations:

$$C_a = \frac{c_a^0}{f},$$ (A20)

$$c_{Yb} = \frac{c_{Yb}^0}{D_{Yb} + f(1 - P_{Yb})},$$ (A21)

$$S = \frac{D_{Yb}}{c_{Yb}^0},$$ (A22)

and

$$I = \frac{(1 - P_{Yb})c_a^0}{c_{Yb}^0}.$$ (A23)

c_{Yb}^0 is assumed to be known, and c_a, c_{Yb}, S, and I can be measured. The four unknowns $(c_a^0, f, D_{Yb}, \text{and } P_{Yb})$ can be obtained from the system of four equations (A20 to A23). It is evident that the resulting values of c_a^0 and f depend on the accuracy of c_{Yb}^0, S and I. Although c_{Yb}^0 (or c_{Lu}^0) varies little, the range of 2.5 to 5.0 times C1 (Frey, 1984; McDonough and Frey, 1989) may still affect the accuracy of c_a^0 and f, which eventually impacts the calculated values for c_b^0, D_b, and P_b. The resulting values for f are probably not as good as those given by the CR method for batch melting, or by the equations presented here for dynamic melting.